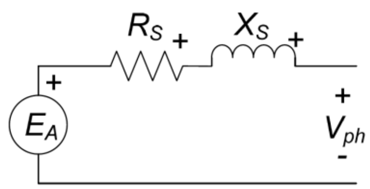
EE3124 Tutorial 5 (Solution)
Synchronous Motor

Name: **Student No.:**

Q1 - A 1000 kVA, 11,000 V, three-phase star-connected synchronous motor has an armature resistance and reactance per phase of 3.5 Ω and 40 Ω respectively.
Draw the vector diagram and determine (a) the voltage drop across armature impedance; (b) the induced EMF and the torque angle of the rotor when it is fully loaded at 0.8 pf lagging.

Solution



Phase voltage: $V_{ph} = \dfrac{11e3}{\sqrt{3}} = 6350\,V$

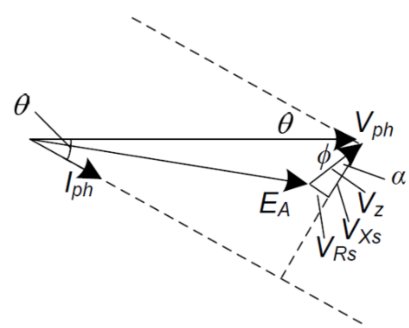
Full load armature current at 0.8 p.f: $\left|I_{ph}\right| = \dfrac{1000e3}{\sqrt{3}11e3} = 52.49\,A$

Calculate the induced e.m.f. and torque angle using KVL and complex number

$$Z_S = \sqrt{3.5^2 + 40^2}\angle \tan^{-1}\frac{40}{3.5} = 40.153\angle 85^o$$

$$\begin{aligned} V_{Zs} &= 40.153\angle 85^o \times 52.49\angle -\cos^{-1}0.8 \\ &= 40.153\angle 85^o \times 52.49\angle -36.87^o \\ &= 2107.63\angle 48.13^o \end{aligned}$$

$$\begin{aligned} E_A &= V_{ph} - V_{Zs} \\ &= 6350 - 2107.63\angle 48.13^o \\ &= 6350 - 1406.72 - j1569.47 \\ &= 4943.28 - j1569.47 \\ &= 5186.45\angle -17.614^o \end{aligned}$$



Q2 - A 2000 V 3-phase, 4-pole, Y-connected synchronous motor runs at 1500 r.p.m.. The excitation is constant and corresponds to an open-circuit terminal voltage of 2000 V. The resistance is negligible as compared with synchronous impedance of 3 Ω per phase. Determine the power input, power factor and torque developed for a lagging armature current of 200 A.

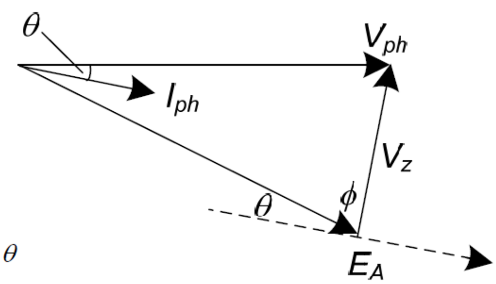
Solution

Phase voltage: $V_{ph} = \dfrac{2e3}{\sqrt{3}} = 1154.70\ \text{V}$

Induced e.m.f.: 1154.70V

Voltage drop across impedance: $V_Z = 200 \times 3 = 600\ \text{V}$

$$V_{ph}^2 = E_A^2 + V_Z^2 - 2E_A V_Z \cos\phi$$
$$1154.7^2 = 1154.7^2 + 600^2 - 2 \times 1154.7 \times 600 \times \cos\phi$$
$$0 = 360000 - 1385640\cos\phi$$
$$\phi = 74.941^\circ$$



$$\theta = 90 - \phi$$
$$= 15.06^\circ$$
$$p.f. = \cos 15.06$$
$$= 0.966$$

$$P_{in} = 3 \times V_{ph} \times I_{ph} \times \cos\theta$$

Input power: $= 3 \times 1154.7 \times 200 \times 0.966$

$$= 669264.12\ \text{W}$$

$$N_S = \frac{r.p.m.}{60\,\text{sec}} = \frac{1500}{60} = 25\ \text{r.p.s.}$$

$$\tau \times \omega = Power$$
$$\tau = 669264.12/(2\pi 25)$$
$$= 4260.67\ \text{N} \cdot \text{m}$$

Q3 - A 480-V, 60 Hz, 400-hp 0.8-PF-leading eight-pole Δ-connected synchronous motor has a synchronous reactance of 0.6 **Ω** and negligible armature resistance. Ignore its friction, windage, and core losses for the purposes of this problem. Assume that $|\mathbf{E}_A|$ is directly proportional to the field current I_F (in other words, assume that the motor operates in the linear part of the magnetization curve), and that $\mathbf{E}_A = 480$ V when $I_F = 4$ A.

(a) What is the speed of this motor?

(b) If this motor is initially supplying 400 hp at 0.8 PF lagging, what are the magnitudes and angles of $\mathbf{E}_A$ and $\mathbf{I}_A$?

(c) According to (b), how much torque is this motor producing? What is the torque angle δ? How near is this value to the maximum possible induced torque of the motor for this field current setting?

(d) If $\mathbf{E}_A$ is increased by 20 percent, what is the new magnitude of the armature current? What is the motor's new power factor?

Solution

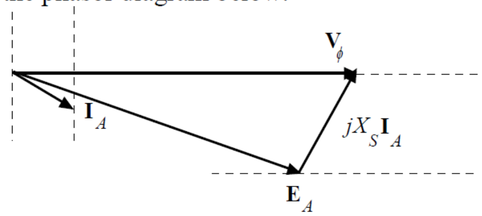
(a) The speed of this motor is given by

$$n_m = \frac{120 f_{se}}{P} = \frac{120(60\text{ Hz})}{8} = 900\text{ r/min}$$

(b) If losses are being ignored, the output power is equal to the input power, so the input power will be

$$P_{\text{IN}} = (400\text{ hp})(746\text{ W/hp}) = 298.4\text{ kW}$$

This situation is shown in the phasor diagram below:



The line current flow under these circumstances is

$$I_L = \frac{P}{\sqrt{3}\,V_T\text{ PF}} = \frac{298.4\text{ kW}}{\sqrt{3}(480\text{ V})(0.8)} = 449\text{ A}$$

Because the motor is Δ-connected, the corresponding phase current is $I_A = 449/\sqrt{3} = 259\text{ A}$. The angle of the current is $-\cos^{-1}(0.80) = -36.87°$, so $\mathbf{I}_A = 259\angle -36.87°\text{ A}$. The internal generated voltage $\mathbf{E}_A$ is

$$\mathbf{E}_A = \mathbf{V}_\phi - jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = (480\angle 0°\text{ V}) - j(0.6\ \Omega)(259\angle -36.87°\text{ A}) = 406\angle -17.8°\text{ V}$$

(c) This motor has 6 poles and an electrical frequency of 60 Hz, so its rotation speed is n_m = 1200 r/min. The induced torque is

$$\tau_{\text{ind}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{298.4 \text{ kW}}{(900 \text{ r/min})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 3166 \text{ N}\cdot\text{m}$$

The maximum possible induced torque for the motor at this field setting is the maximum possible power divided by ω_m

$$\tau_{\text{ind,max}} = \frac{3V_\phi E_A}{\omega_m X_S} = \frac{3(480 \text{ V})(406 \text{ V})}{(900 \text{ r/min})\left(\dfrac{1 \text{ min}}{60 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1 \text{ r}}\right)(0.6\ \Omega)} = 10,340 \text{ N}\cdot\text{m}$$

The current operating torque is about 1/3 of the maximum possible torque.

(d) If the magnitude of the internal generated voltage E_A is increased by 20%, the new torque angle can be found from the fact that $E_A \sin\delta \propto P = \text{constant}$.

$$E_{A2} = 1.20\, E_{A1} = 1.20(406 \text{ V}) = 487.2 \text{ V}$$

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_1\right) = \sin^{-1}\left(\frac{406 \text{ V}}{487.2 \text{ V}}\sin(-17.8°)\right) = -14.8°$$

The new armature current is

$$\mathbf{I}_{A2} = \frac{\mathbf{V}_\phi - \mathbf{E}_{A2}}{jX_S} = \frac{480\angle 0° \text{ V} - 487.2\angle -14.8° \text{ V}}{j0.6\ \Omega} = 208\angle -4.1° \text{ A}$$

The magnitude of the armature current is 208 A, and the power factor is cos (· !4.1°) = 0.913 lagging.